

VARIABILITY OF LOW-LEVEL MOISTURE FLUX AND ITS RELATION WITH PRECIPITATION

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Abstract: This study investigates long-term variation of low-level vertically integrated moisture flux (VIMF) from 1000 hPa to 850 hPa for 37-year from 1979 to 2015 and its contributions to the total rainfall over South Asia (SA), which includes also the Bay of Bengal and the eastern part of Arabian Sea. ERA-Interim 6-hourly reanalysis data generated by European Centre for Medium-Range Weather Forecasts (ECMWF) having horizontal resolution of approximately 80 km are used in this study. The details of spatial and temporal characteristics of low-level moisture flux and its effect on precipitation are analyzed for the sub-regions of South-West (SW), Bay of Bengal (BoB), South-East (SE), North-West (NW), Bangladesh and surroundings (BAN), and North-East (NE). The average value of 37 years VIMF over the study area is found 113.71 kg m⁻¹ s⁻¹, which remains almost constant over the year. About 41% of the total VIMF is found to be advected in the monsoon season, whereas, in the winter season only 17% is transported. The value of VIMF is highest (191.74 kg m⁻¹ s⁻¹) in July over SA which is 33% more than the minimum value in March. For six sub-domains, 27% of the total VIMF is found over the BoB whereas only 6% is found over the NE domain. The SW, SE, NW and BAN regions receive 21%, 19%, 13% and 14% of the total flux, respectively. Southern part of the study area (SW, BoB and SE) acquires 67% of the total VIMF, whereas, 33% is found over the Northern part (NW, BAN and NE). Seasonal variation indicates that the Southern parts of the study area have two times higher VIMF than the Northern parts in the monsoon season. Low level moisture flux contributes large amount of precipitation over the study area. 72% of total precipitation is coming from moisture transport whereas only 28% contributes from local evaporation. Individual domains also show the similar contributions. The coefficient of determination (correlation coefficient) between VIMF and the advective component of precipitation is 0.87 (0.96) and that for evaporation and the precipitation arising from local evaporation (P_m) is 0.456 (0.85).

Keywords: Vertically Integrated Moisture flux, Precipitation, Evaporation, South Asia.

1. INTRODUCTION

Among the various climatic parameters which influence the growth of crops in South Asia, the most important parameter is considered to be precipitation. Precipitation is essential because it helps to maintain the atmospheric balance. It supplies freshwater to estuaries, which is a fundamental source of dissolved oxygen and nutrients. It also plays an important role in the economy and economical growth of South Asia directly or indirectly. Basically, precipitation depends on amount of water vapour in the atmosphere, which comes from both local evapo-transpiration (ET) and remote moisture transport or moisture flux [1, 2]. Usually, most of the precipitation comes from the moisture flux, even though exact amount of precipitation from moisture flux or from evaporation are crucial to enrich the knowledge about precipitation over South Asia.

Most of the rainfall is produced from convective cloud systems in which rainfall intensity varies through the life time of the cloud system. The formation and longevity of convective system mainly depends on the amount of moisture availability at low level. As a result, low level moisture flux or moisture flux convergence is considered as a useful tool in prediction of convection as well as precipitation [3, 4]. Many previous studies have used vertically integrated moisture flux to estimate the moisture sources [5-7]. Trenberth (1998) has explained the moisture quantity that precipitates out comes from local evaporation versus horizontal transport [8]. Leslie et al. (2015) has reported that the total low-level moisture flux contributes to rainfall over West Africa and also showed that the correlation between components of moisture transport and precipitation confirm the strong association between the zonal moisture transport and Sahelian precipitation [3].

Very few studies have investigated the moisture transport over South Asia (SA) using reanalysis data and modeling [9-11]. They mainly analyzed moisture flux associated with the Indian rainfall in monsoon. Huang et al. (2015) has showed that strong moisture flux started from the Arabian Sea and was then transported the Indian Peninsula and Bay of Bengal [10]. Karmakar, S. (1998) has used vertically integrated tropospheric moisture to find out the characteristic of three major cyclones at Bangladesh coast. He showed that vertically integrated

tropospheric moisture has a tendency to decrease the cyclones in the Bay of Bengal at the formation stages and significant increase as the cyclones move northwards for ultimate landfall [11]. However, to our knowledge no study has been done for the low-level VIMF and its long-term variability over SA including the Bay of Bengal. Therefore, it is essential to know the distribution of low-level VIMF in synoptic scale and associated rainfall over South Asia and the Bay of Bengal. Therefore, the purpose of this paper is to analyze the variations of moisture transport and its relation to precipitation that will help to better understand the large-scale atmospheric circulation, atmospheric moisture availability and precipitation over South Asia.

2. DATA AND METHODS

Global atmospheric 6-hourly reanalysis data product ERA-Interim generated by European Centre for Medium-Range Weather Forecasts (ECMWF) [5] were used in this study for 37 years from 1979 to 2015. The horizontal resolution of data is approximately 80 km with 60 vertical levels. Data for meridional and zonal wind components and specific humidity at different pressure levels were utilized for analyzing low level vertically integrated moisture flux (VIMF) for South Asia including the Bay of Bengal (BoB) and eastern part of the Arabian Sea (5°N-27°N & 70°E-105°E) shown in Fig. 1.

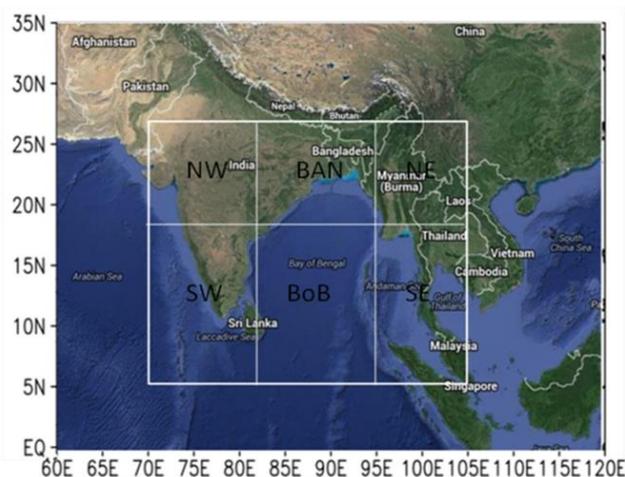


Fig. 1: Total study area and six sub-regions.

The VIMF is defined as
$$\frac{1}{g} \int_{p_s}^p qV dp$$

Where V the horizontal wind, g is the gravitational acceleration and q is the specific humidity [3, 4]. For the purpose of this work, VIMF was calculated for the pressure levels of 1000 hPa to 850 hPa for understanding spatial and temporal characteristics of moisture flux and its consequence on precipitation in the study region. The total study area is further divided into six sub-regions i.e. South-West (SW), Bay of Bengal (BoB), South-East (SE), North-East (NE), Bangladesh and surrounding (BAN) and North-West (NW) region (Fig. 1). Six domains are divided by centering the BoB, which is the main source of moisture and transports to the surrounding land including Bangladesh.

In this study, a simple method according to Trenberth (1998) are calculated for differentiating precipitation that comes from the moisture advection and local evaporated moisture [6]. The process is as follows:

Precipitation by evaporation,

$$P_m = \frac{EL}{EL + 2F_{in}} \times P \quad (1)$$

Precipitation by advection,

$$P_a = P - P_m = \left(1 - \frac{EL}{EL + 2F_{in}}\right) P \quad (2)$$

Where, P is precipitation, P_a is the advective components of precipitation, P_m means precipitation comes from local evaporation, F_{in} is moisture flux that enters into the area, E is Evaporation, and L is the length of the considerable domain.

3. RESULTS AND DISCUSSION

The distributions of 37-year averaged VIMF and horizontal wind are analyzed for the study area during 1979-2015 shown in Fig. 2. Total VIMF over the region is calculated to be $113.71 \text{ kg m}^{-1} \text{ s}^{-1}$. Highest amount of moisture, about 47% of the total moisture flux, is found over the BoB and the eastern part of AS, mainly the Ocean area which contributes a significant amount of moisture to the considerable area. Maximum value of VIMF is found above $100 \text{ kg m}^{-1} \text{ s}^{-1}$ over the BoB and above $90 \text{ kg m}^{-1} \text{ s}^{-1}$ for the South-West part of the study area. More than $70 \text{ kg m}^{-1} \text{ s}^{-1}$ VIMF is observed over the South-East domain. The value of VIMF over Bangladesh and the North-West part is above $70 \text{ kg m}^{-1} \text{ s}^{-1}$ and $75 \text{ kg m}^{-1} \text{ s}^{-1}$ respectively. Lowest amount of moisture flux is found over the North-East part which is only $20 \text{ kg m}^{-1} \text{ s}^{-1}$.

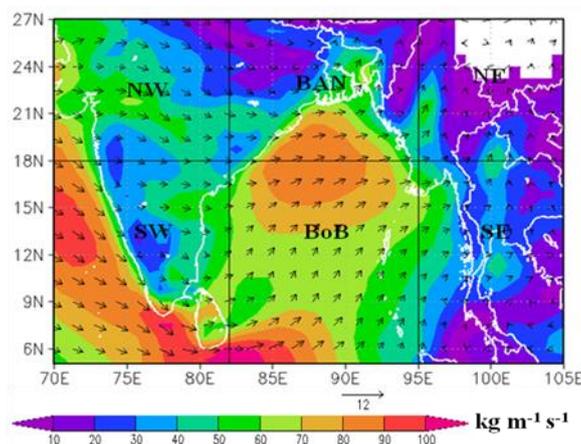


Fig. 2: Spatial distribution of average VIMF (shading; $\text{kg m}^{-1} \text{ s}^{-1}$) and average wind velocity (vector; m s^{-1}) from 1000 hPa to 850 hPa during 1979-2015.

3.1 Annual variation of VIMF

The annual distribution and the 5-year running mean of 37-year averaged VIMF over the total study area are shown in Fig. 3 which fluctuates little from year to year (1979 to 2015). Maximum value of moisture flux is observed to be $124.33 \text{ kg m}^{-1} \text{ s}^{-1}$ in 1990 and minimum value is observed to be $107.99 \text{ kg m}^{-1} \text{ s}^{-1}$ in 1997. Though there is little variation from year to year but the annual variation of VIMF remains constant for 37 years over SA with co-efficient of determinant (R^2 value) of 0.005. Average increasing rate is only 0.14% per year.

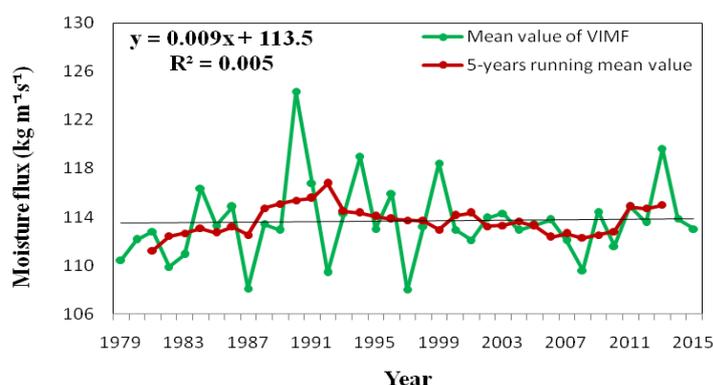


Fig. 3: Yearly variation of average VIMF (green line) for the study area. Red line indicates the 5-year running mean from 1979 to 2015.

3.2 Seasonal variation of VIMF

Spatial distribution of 37-year averaged low-level VIMF for the pre-monsoon (March-May), monsoon (June-September), post-monsoon (October-December) and winter (January-February) seasons are shown in Fig. 4. Here, seasons are divided according to Indian Meteorological Department [12]. Amount of moisture fluxes in the pre-monsoon and post-monsoon seasons are relatively less than that in the monsoon season which shows the highest amount of VIMF. In the monsoon season, huge amount of moisture is carried by the strong southwesterly monsoonal wind. Wind helps to propagate moisture north-northeastward in the pre-monsoon,

while in the summer monsoon strong southwesterly winds transport moisture from southwest to northeast. The wind intensity in summer monsoon is much stronger than the winter monsoon. Wind intensity reaches its peak in July-August and fades away in October-November. Wind direction is completely reverse in the post-monsoon and the winter seasons which carry less moisture from the land to ocean. However, strong easterly winds convergence near Borneo and carry moisture to the BoB in the winter season which is consistent with Akter & Tsuboki (2012) [13].

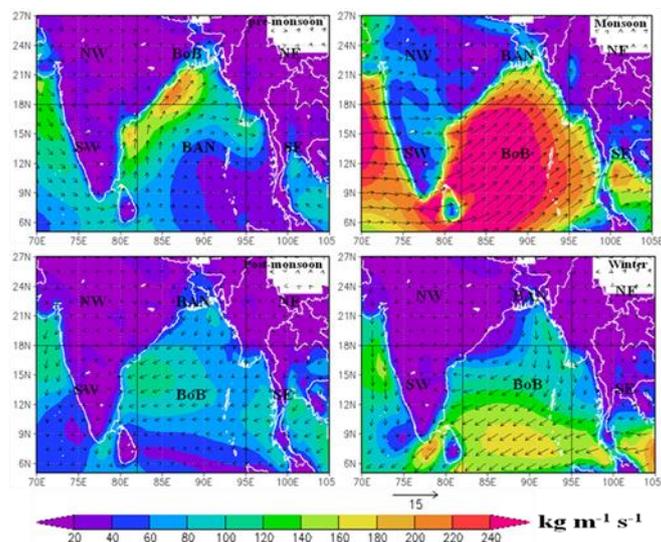


Fig. 4: Spatial distribution of average VIMF (shading; $\text{kg m}^{-1} \text{s}^{-1}$) and average wind velocity (m s^{-1}) for 1000-850 hPa in different seasons during 1979-2015.

The quantities of average VIMF over this region is shown in Fig. 5. Highest amount of VIMF is found in the monsoon season; approximately 41% of the total moisture flux is found to be advected in the monsoon season. About 21% of the total moisture flux is observed in the pre-monsoon season that is approximately same as the post-monsoon season. Only 17% of the total VIMF is observed in the winter season.

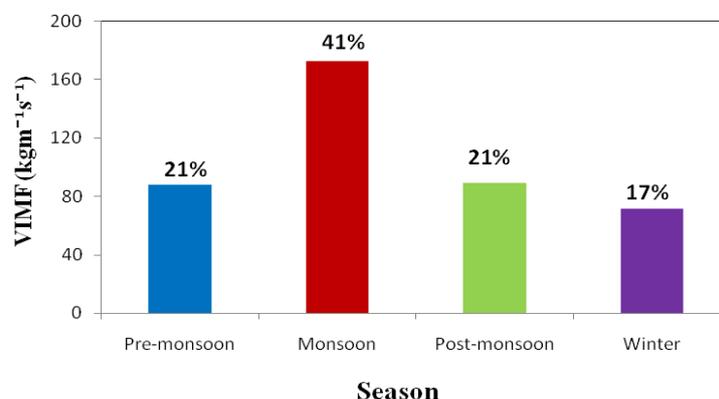


Fig. 5: Seasonal variation of average VIMF during 1979-2015.

The average seasonal increasing rate of vertically integrated moisture flux per year is approximately 0.5% both in the pre-monsoon and winter seasons but for the monsoon and post-monsoon seasons, these amounts are 0.31% and 0.33%, respectively, which are almost constant.

3.3 Monthly variation of VIMF

Monthly variation of low-level average VIMF between 1000 hPa to 850 hPa for South Asian region is calculated and shown in Fig. 6. Maximum value of VIMF is observed $191.74 \text{ kg m}^{-1} \text{ s}^{-1}$ in July and minimum value is found $63.74 \text{ kg m}^{-1} \text{ s}^{-1}$ in March which is 33% less than the maximum value. At the beginning of April, moisture flux is increasing, peak in July and then decreasing gradually from October to December and these three months show quite similar values of moisture flux. Mainly a large amount of moisture flux is found in June, July, August and September over the study area.

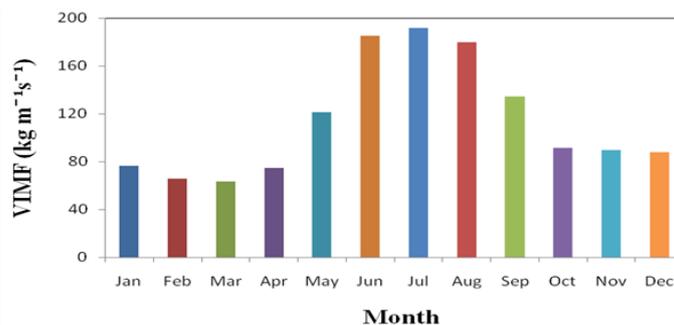


Fig. 6: Monthly variation of average VIMF for total domain during 1979-2015.

3.4 Variation of VIMF for six domains

Yearly variation of 5-year running mean value of low-level 37-year averaged VIMF for six domains are shown in Fig. 7. Very little variation is found for all domains in the years of 1991-1995 except North-East domain in which VIMF is increased with R^2 of 0.54. Maximum value of VIMF is found over the Bay of Bengal and lowest amount of VIMF is observed over the North-East domain. About 27% of the total moisture flux is found over the Bay of Bengal. 21% of the total moisture flux is found over the South-West domain, 19% of the total flux over the South-East domain and 13% of the total flux over the North-West domain and 14% of the total over Bangladesh & surrounding. Only 6% of the total moisture flux is observed over the North-East domain. Southern part of the study area acquires 67% of the total VIMF, whereas, 33% of the total VIMF is found over the Northern part.

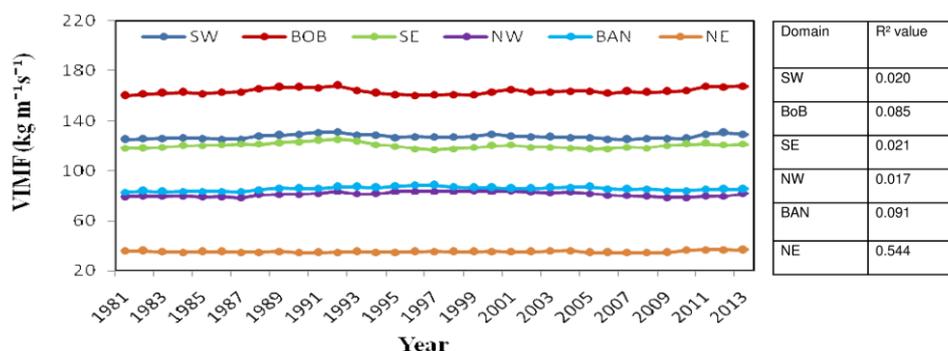


Fig. 7: 5-year running mean of average VIMF for six domains (1979-2015).

Seasonal and monthly variations of averaged low-level VIMF in Fig. 8a and 8b indicates that the monsoon season transports highest amount of VIMF for all domains and having more moisture the Southern regions (SW, BoB and SE) than that in the Northern regions (NW, BAN and NE) during the pre-monsoon, post-monsoon and the winter seasons. Moisture transport is highest in July for all study areas, however, in the BoB it is little higher in June and for the South-East domain it is high in the month of August.

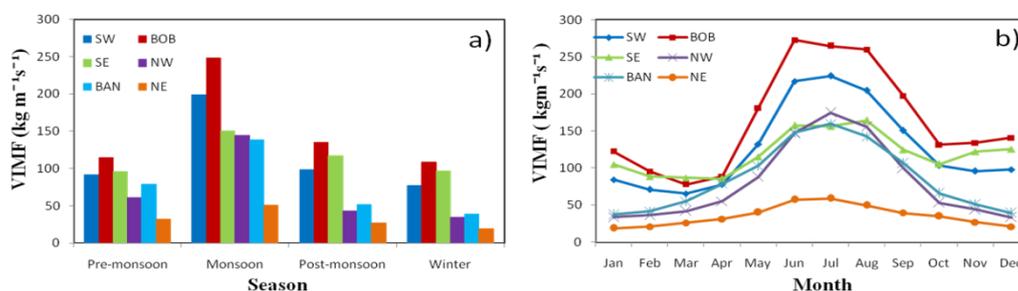


Fig. 8: (a) Seasonal and (b) monthly variations of average VIMF for six domains during 1979-2015.

For all domains lowest amount of VIMF is found in the month of January for Northern region and March for the Southern region. Maximum value is found in July (on average $154.40 \text{ kg m}^{-1} \text{ s}^{-1}$) for all domains except BoB (June, $272.86 \text{ kg m}^{-1} \text{ s}^{-1}$) & SE (August, $164.19 \text{ kg m}^{-1} \text{ s}^{-1}$) whereas minimum value is found in January (on average $29.73 \text{ kg m}^{-1} \text{ s}^{-1}$) for Northern region and March for (on average $76.78 \text{ kg m}^{-1} \text{ s}^{-1}$) Southern region. Bangladesh and surrounding has higher flux than the North-West and the North-East part for all seasons but in the monsoon season the North-West part has higher moisture flux than Bangladesh and surrounding. Fig. 10

shows that for all domains the highest moisture flux is observed in the monsoon season and lowest value in the winter season. VIMF is observed about 42%, 41%, 33%, 51%, 45% and 39% in the monsoon season for SW, BoB, SE, NW, BAN and NE domains, respectively, whereas, in the winter season moisture flux are 17%, 18%, 21%, 12%, 12% and 15%. The post-monsoon season has higher moisture flux than the pre-monsoon season for the SW, BoB and SE domains but opposite for the NW, BAN and NE domains.

3.5 Calculation of precipitation associated with moisture flux

For calculating precipitation that comes from moisture advection or comes from evaporation are analyzed for the study period. The relation between VIMF and total precipitation (P), total evaporation (E) and P are shown by scatter plots in Fig. 9. The co-efficient of determination (R^2 value) between precipitation and moisture flux for the study area shows the significant value of 0.77 which means 77% of the total variation in precipitation can be explained by the linear relationship between them. Whereas, R^2 value between evaporation and precipitation is 0.23 i.e. only 23% of the total variation of precipitation can be explained by the relationship between E and P over the study area. Therefore, moisture flux plays an important role to form precipitation over the study area rather than evaporation.

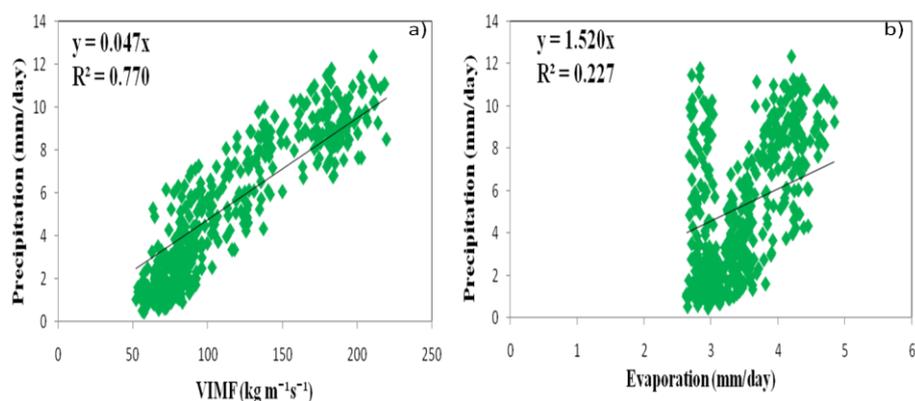


Fig. 9: Scatter plots between (a) VIMF and P and (b) E and P for the study area during 1979-2015.

Using equation 1 and 2, precipitation advected by moisture flux (P_a) and precipitation arising from local evaporation (P_m) are calculated and shown in Table-1 along with the 37-year averaged VIMF, E and P for six domains and total domain.

Table-1: Value of VIMF, E, P, P_a and P_m for six domains.

Domain	VIMF ($\text{kg m}^{-1} \text{s}^{-1}$)	E (mm/day)	P (mm/day)	P_a (mm/day)	P_m (mm/day)
SW	127.48	3.33	4.03	3.34	0.69
BoB	164.28	4.75	6.34	5.16	1.18
SE	119.96	4.92	6.35	4.74	1.61
NW	81.00	1.28	1.77	1.64	0.13
BAN	85.46	2.90	4.96	4.25	0.71
NE	35.01	3.71	7.07	4.47	2.60
TOTAL	113.71	3.45	5.09	3.65	1.44

The distributions of advective components of precipitation (P_a) associated with VIMF and the precipitation arising from local evaporation (P_m) are shown in Fig. 10. Advective component of precipitation is about 72% of the total precipitation and 28% of the total precipitation arises from local evaporation.

For all domains, advective precipitation is higher than the precipitation arising from local evaporation. The component of P_a over the BoB domain (5.16 mm/day) is higher than the other domains, whereas, P_m is highest (2.6 mm/day) over the NE domain. Advective precipitation is five times greater than P_m for the SW, BoB and BAN region. For the NE domain, P_a (4.47 mm/day) is 1.72 times greater than P_m (2.6 mm/day). Both moisture flux and evaporation play important role to form precipitation over the NE domain and highest amount of precipitation (7.07 mm/day) is found over this region.

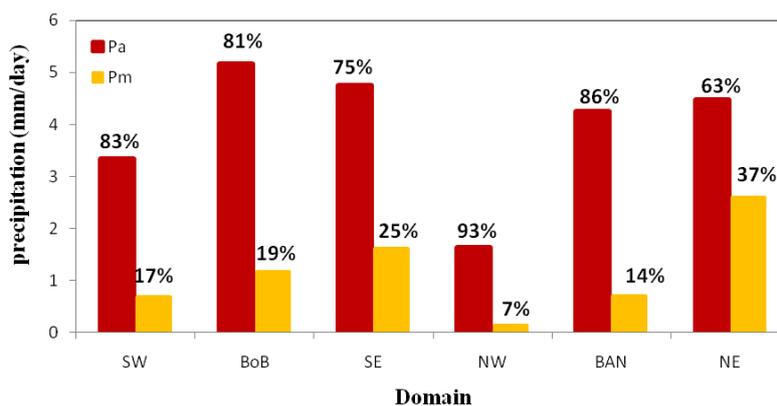


Fig. 10: Percentage of the advective component of precipitation (P_a) and precipitation arising from local evaporation (P_m) for six domains.

The co-efficient of determination (R^2 value) and correlation coefficient (r value) between VIMF and P_a , E and P_m for six domains and total domain are given in Table-2. For all domains, the correlation and R^2 values between E and P_m are found approximately 0.76 and 0.58 on average, respectively except the SW ($r=0.405$, $R^2=0.164$) and BoB ($r=0.213$, $R^2=0.045$) domains which have comparatively low values. The inconsistency in P_m over these areas, especially over ocean, may be caused by neglecting the contribution of the existing moisture in the above mention equation (1 and 2). The result will be more accurate for P_m if the precipitation for existing moisture over the ocean can be calculated.

Table-2: Monthly average R^2 value and r value for six domains (1979-2015)

Domain	R^2		r	
	VIMF and P_a	E. and P_m	VIMF and P_a	E and P_m
SW	0.714	0.164	0.845	0.405
BoB	0.652	0.045	0.808	0.213
SE	0.401	0.423	0.633	0.650
NW	0.716	0.608	0.846	0.779
BAN	0.864	0.620	0.929	0.787
NE	0.896	0.668	0.946	0.817
Total Domain	0.871	0.456	0.933	0.675

6. CONCLUSIONS

Precipitation is the most important atmospheric parameter. To analyze the cause of precipitation and its amount, low-level vertically integrated moisture flux (VIMF) from 1000 to 850 hPa is considered and calculated in this study. Long-term annual, seasonal and monthly variability of VIMF are measured over this region and find out their relations with precipitation. The average value of VIMF over the study area is found 113.71 kg m⁻¹ s⁻¹, which has the average annual increasing rate of 0.14% per year. Southern part of the study area including ocean area acquires 67% of the total flux, whereas, 33% of the total moisture flux is found over the Northern part, mainly the land area. According to the season, the highest (lowest) amount of VIMF is found 41% (17%) of the total VIMF in the monsoon (winter) season, whereas, other two seasons have 21% of the total VIMF for each. The 37-year averaged VIMF over total study area maintains a periodic pattern throughout the twelve months of the year with a maximum value of 191.74 kg m⁻¹ s⁻¹ in July and the minimum value of 63.74 kg m⁻¹ s⁻¹ in March which is 33% less than the maximum value. The intense southwesterly wind in the monsoon season transports the highest amount of moisture for all domains. Within the domains, the Southern regions (SW, BoB and SE) have two times higher VIMF than the Northern regions (NW, BAN and NE) in the monsoon season. In the Northern regions, BAN has more flux than the other domains for all seasons but in the monsoon season where the NW part has higher VIMF. Monthly variation of 37-year averaged VIMF shows the maximum value in July for all domains except the BoB and SE domains which have maximum value in the month of June and August, respectively. The lowest amount of VIMF is observed approximately 29.73 kg m⁻¹ s⁻¹ in January for the Northern regions and that in March approximately 76.78 kg m⁻¹ s⁻¹ for the Southern regions of the study area.

According to the equation used by Trenberth (1998), the total precipitation (P) over SA is separated into two components i.e. the advective component of precipitation (P_a) and the precipitation arising from local evaporation (P_m) and finally precipitation components are correlated with VIMF and evaporation (E) values.

The coefficient of determination (correlation coefficient) between VIMF and is 0.871 (0.933) and that for evaporation is 0.456 (0.675) over the total domain. For all domains, advective precipitation is higher than the precipitation that comes from local evaporation that are 83%, 81%, 75%, 93%, 86%, 63% and 17%, 19%, 25%, 7%, 14%, 37% for the SW, BoB, SE, NW, BAN and NE domains, respectively. For all domains, the correlation and R^2 values between E and are found approximately 0.76 and 0.58 on average, respectively except the SW ($r=0.405$, $R^2=0.164$) and BoB ($r=0.213$, $R^2=0.045$) domains which have comparatively low values. Neglecting the contribution of the existing moisture in the above mention equation can cause the inconsistency over SA, especially over ocean. Therefore, further study by including the contribution of the existing moisture over SA will help the researchers to know the accurate contributions to the precipitation.

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